COMMENTS

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Comment on ''Universal formulas for percolation thresholds''

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(Received 17 June 1996)

Recently, Galam and Mauger postulated a power law for both site and bond percolation thresholds, based on a fit to exact and numerical values of the thresholds [Galam and Mauger, Phys. Rev. E **53**, 2177 (1996)]. The power law predicts percolation thresholds, based solely on the dimension d and the coordination number q of the network. However, I give an example of two networks, where d and q are equal, but the percolation thresholds differ. [S1063-651X(97)06001-7]

PACS number(s): 64.60.Ak, 64.60.Cn, 64.70.Pf

In a recent paper, Galam and Mauger postulated a power law for both site and bond percolation thresholds [1]. The basis for their postulate is a remarkably good fit to exact and numerical values of percolation thresholds. The power law enables one to calculate the percolation threshold p_c of a network, with the dimension d and the coordination number q of the network as the only input. The power law reads

$$p_c = p_0[(d-1)(q-1)]^{-a} d^b.$$

The parameters p_0, a, and b are determined by a fit to known percolation thresholds.

However, there are examples of networks with equal d and q, but with different percolation thresholds. Therefore it seems likely that either there are more universality classes than introduced by Galam and Mauger, or, if there exists a universal formula for percolation thresholds, it needs to be based on more information than d and q only.

To demonstrate that d and q are not sufficient to determine the percolation threshold of a system, I present the example of the body centered cubic (bcc) lattice and the stacked triangular lattice (sometimes called simple hexagonal lattice [2]). Both lattices have $d=3$ and $q=8$. Nevertheless their percolation thresholds differ: 0.246 vs 0.2623 for site percolation and 0.1803 vs 0.1859 for bond percolation. The values for the bcc lattice are taken from Stauffer and Aharony [3], the ones for the stacked triangular lattice I calculated myself.

So there are networks with equal dimension and coordination number, but with different percolation thresholds. It is of course possible that these networks belong to different universality classes. However, so far, all the three-dimensional networks that were included in the study of Galam and Mauger belonged to the same universality class. It would deprive the concept of universal formulas of its elegance if we would have to introduce more universality classes. On the other hand, the stacked triangular lattice is the only anisotropic lattice that is considered in the framework of the universal formulas. Therefore my conclusion is that either there needs to be more universality classes, or a universal formula for percolation thresholds, if it exists, will

TABLE I. The percolation thresholds for the stacked triangular lattice, as a function of the linear system size L. The site percolation threshold is listed for several directions separately. The notation xy indicates that the cluster algorithm searched for spanning clusters in both the x and y direction. Between parentheses are error estimates concerning the last digit. The values for $L=\infty$ are results of a fit of the scaling relation to the last three data points.

L	site, xy		site, z		site, xyz		bond, xyz	
16	0.2538	(2)	0.2731	(2)	0.2569	(2)	0.1837	(2)
32	0.2575	(2)	0.2673	(2)	0.2595	(2)	0.1846	(2)
64	0.2598	(2)	0.2644	(2)	0.2609	(2)	0.1852	(2)
128	0.2612	(2)	0.2635	(2)	0.2616	(2)	0.1857	(2)
250	0.2618	(2)	0.2627	(2)	0.2620	(2)		
∞	0.2623	(2)	0.2624	(2)	0.2623	(2)	0.1859	(2)

have to be based on more than dimension and coordination number only.

For the calculation of percolation thresholds, I used the method outlined by Stauffer and Aharony [3], p. 73. In the binary search for the percolation threshold of each particular network realization, I took 16 steps, to have sufficient accuracy. The random number generator I employed was taken from Marsaglia, Zaman, and Tsang [4]. Since the stacked triangular lattice is anisotropic, I calculated the percolation threshold in several directions separately. In all other directions I applied periodic boundary conditions. The results are shown in Table I, for networks of various sizes. The variable L denotes the linear system size. The system size dependent results $p_c(L)$ can be fitted to the scaling relation

$$|p_c(L)-p_c(\infty)|\sim L^{-1/\nu}.$$

Here, ν is a critical exponent, which is kept fixed in the fitting procedure, at $\nu=0.88$ in three dimensions [3]. The result of the fit is an estimate of $p_c(\infty)$, which is also listed in Table I. As a check on my program, I ran the program for the bcc lattice as well. My result for the bcc lattice is 0.2458 ± 0.0002, which is consistent with the value reported by Stauffer and Aharony [3].

[1] S. Galam and A. Mauger, Phys. Rev. E **53**, 2177 (1996).
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[3] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, London, 1992).
[4] G. Marsaglia, A. Zaman, and W.W. Tsang, Stat. Prob. Lett. **8**, 35 (1990); F. James, Comput. Phys. Commun. **60**, 329 (1990).